

**Dhirubhai Ambani Institute of
Information and Communication Technology (DA-ICT)
Ph.D. Entrance Examination
(Sample Paper)**

Name:

Application No:

INSTRUCTIONS:

- This examination has 5 sections viz. i) Mathematics, ii) Physics, iii) Communication and Signal Processing, iv) Computer Science, and v) Electronics.
- Write your name and application number clearly in the space provided at the top of main page. The booklet has to be returned at the end of the examination.
- All questions carry equal marks. Answer as many questions as you can. This may be done across any number of sections. Use a separate answer booklet for answering each section. Write your name, application number, section number, and section title on the first page of each answer booklet that you use. You have two hours to complete the whole test.

FOR EVALUATOR'S USE ONLY

Question No	Mathematics	Physics	Communication and Signal Processing	Computer Science	Electronics
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
Total					
PART A					

PhD Entrance Written Test Mathematics

1. Suppose that the travel time from your home to office is normally distributed with mean 40 minutes and standard deviation 7 minutes. If you want to be 95% sure that you will not be late for an office appointment at 1 p.m., what is the latest time you should leave home. (Given : $\Phi(1.64) = 0.95$).

2. Let $T: V \rightarrow V$ be a linear operator. A subspace $S \subseteq V$ is called an invariant subspace of T if $T(s) \subseteq S, \forall s \in S$. Show the following are invariant subspaces of T .
 - (i) $I(T) = \{T(v): v \in V\}$

 - (ii) $N(T) = \{v \in V: T(v) = 0\}$

 - (iii) $N(T - \alpha I)$, α is any complex number.

3. Show that every subgroup of a cyclic group is cyclic.

4. A secretary is carrying 10 envelopes with letters. He/She drops the envelopes and then places the letters randomly back in the envelopes. Find the number of ways that he/she can place the letters such that none of the original letters gets placed in its correct envelope.

5. Solve the following system of linear differential equations
$$\frac{dx}{dt} + 4x + 3y = t$$
$$\frac{dy}{dt} + 2x + 5y = e^t$$

6. Show that the largest rectangle inscribed in a circle is a square.

7. Let a function $f: [a, b] \rightarrow R$ be continuous on $[a, b]$ and differentiable on (a, b) . If $f'(x) = 0$ for all $x \in (a, b)$, then prove that f is a constant on $[a, b]$.

8. Construct an analytic function $f(z)$ whose real part is $e^x \cos y$.

PhD Entrance Written Test Physics

$g=9.8 \text{ m/s}^2$,
permittivity of free space = $8.85 \times 10^{-12} \text{ [F/m]}$
Mass of proton = $m_p = 1.673 \times 10^{-27} \text{ kg} = 1.007277 \text{ u}$
Mass of neutron = $m_n = 1.675 \times 10^{-27} \text{ kg} = 1.008665 \text{ u}$
Mass of electron = $m_e = 9.109 \times 10^{-31} \text{ kg} = 0.00055 \text{ u}$
Mass of hydrogen atom = $m_H = 1.007825 \text{ u}$
 $1 \text{ u} = 931 \text{ MeV}$.
Charge of electron = $1.60217657 \times 10^{-19} \text{ coulombs}$
 $1 \text{ G} = .0001 \text{ Tesla}$.
Planck's constant = $h = 6.626 \times 10^{-34} \text{ joule-sec}$
Speed of light = $c = 3 \times 10^8 \text{ m/sec}$
 $1 \text{ eV} = 1.6 \times 10^{-19} \text{ joule}$
Boltzmann's constant = $k = 1.38 \times 10^{-23} \text{ joule}$

Mechanics

- 1) Show that if the total linear momentum of a system of particles is zero, the angular momentum of the system is the same about all origins.
- 2) Two particles have masses m_1 and m_2 . m_1 exerts a force \vec{f}_{12} on m_2 while m_2 exerts a force \vec{f}_{21} on m_1 . External forces \vec{F}_1 and \vec{F}_2 act on m_1 and m_2 respectively. Justify that the acceleration of the center of mass of the system of two particles is $\frac{\vec{F}_1 + \vec{F}_2}{m_1 + m_2}$.

Mathematical Physics

- 3) Verify the divergence theorem for a vector field $\vec{F}(\vec{r}) = \vec{r}$ over a spherical volume of radius 1 with its center at the origin of the coordinate system.
- 4) (a) Suppose $\vec{V}(x, y, z) = (x - y)\hat{i} + (x + y)\hat{j}$. Compute $\vec{\nabla} \cdot \vec{V}$ and $\vec{\nabla} \times \vec{V}$.

(b) If \vec{W} is a vector field and ϕ is a scalar field, indicate whether each of the five following expressions is a vector field, a scalar field, or doesn't make sense.
 - (i) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{W})$
 - (ii) $\vec{\nabla} \times (\vec{\nabla} \cdot \vec{W})$
 - (iii) $\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{W})$
 - (iv) $\vec{\nabla} \times (\vec{\nabla} \times \vec{\nabla} \phi)$
 - (v) $\vec{\nabla} \cdot \vec{\nabla} \phi - \vec{\nabla}(\vec{\nabla} \cdot \phi)$

Electromagnetic Theory

- 5) A point charge q is placed at a distance r from the center of a spherical conducting shell of radius a . What is the potential of the shell if
 - (i) $r < a$
 - (ii) $r > a$

- 6) (a) A uniform magnetic field $B=3.0$ G exists in the $+x$ direction. A proton shoots through the field in the $+y$ direction with a speed of 5×10^6 m/s. Find the magnitude and direction of the force on the proton.
- (b) The electric field \vec{E} in free space is given as $\vec{E} = E_0 \cos(\omega t - kz) \hat{i}$. Determine \vec{D} , \vec{B} and \vec{H} .

Quantum Mechanics

- 7) A quantum mechanical system consists of a particle of mass m in a two dimensional potential well $V(x, y)$ given as

$$V(x, y) = 0; \text{ when } 0 < x < L, 0 < y < L$$

$$= \infty \text{ otherwise}$$

The particle is in a stationary state described by the wavefunction

$$\Psi(x, y) = \frac{\sqrt{2}}{L} \left(\sin \frac{\pi x}{L} \sin \frac{2\pi y}{L} + \sin \frac{\pi y}{L} \sin \frac{2\pi x}{L} \right); \text{ when } 0 < x < L, 0 < y < L$$

$$= 0 \text{ otherwise}$$

Justify that this state is an eigenstate of the Hamiltonian of the system. Find the energy of the particle in this state.

- 8) In a photoelectric effect experiment when light of wavelength 3000 Å used, the current goes to zero when a stopping potential of 2.65V is applied. Whereas when the wavelength of the light is 5000 Å the stopping potential required is 1V. Find the Planck's constant h from this data.

Thermodynamics

- 9) (a) In some process, 7 kcal of heat is furnished to the system while the system does 5 kJ of work. Calculate by how much does the internal energy of the system change during the process? (1 joule = 0.239005736 calories).
- (b) The Helmholtz thermodynamic Potential F is defined by $F=U-TS$. Derive the Maxwell Relation associated with the Helmholtz Potential.

Nuclear Physics

- 10) (a) The atomic mass of $^{16}_8\text{O}$ is 15.9949 u. What is its binding energy?

(b) When ${}^{235}_{92}\text{U}$ undergoes fission, about 0.1 percent of the original mass is released as energy. How much energy is released when 2 kg of ${}^{235}_{92}\text{U}$ undergoes fission?

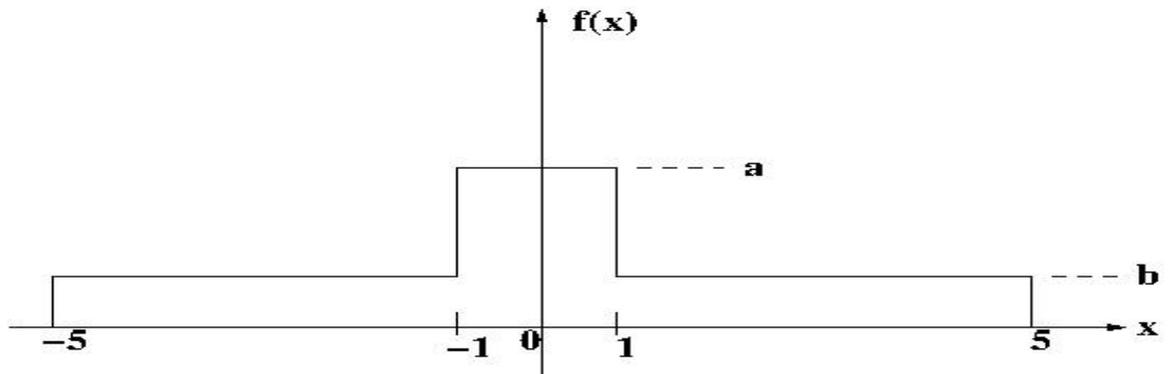
(c) The half life of the sodium isotope ${}^{24}_{11}\text{Na}$ against beta decay is 15 Hours. How long will it take for seven-eighths ($7/8$) of a sample of this isotope to decay?

PhD Entrance Written Test

Communications and Signal Processing

Analog and Digital Communications

1. A sinusoid of frequency 5 kHz modulates a sinusoidal carrier of frequency 1 MHz to produce a DSB signal. The percentage of modulation used is 50. The modulated signal is then passed through a channel whose frequency response is similar to that of a resonant circuit centered at 1 MHz and with a Q factor of 95. The signal received from the channel is passed through an envelope detector. Can the envelope detector demodulate the received signal correctly? Justify. (10 marks)
2. An input to a 6-level quantizer has the probability density function $f(x)$ as shown in the figure below. The decision boundaries are chosen so that the quantizer output is equiprobable. It is further given that 3 consecutive decision boundaries are -1, 0 and 1. Given this (i) determine the value of a and b given in the figure (6 marks) and (ii) compute the remaining 3 decision boundaries (4 marks).



Electromagnetic Theory

1. (a) Explain 4 main application areas of microwaves? (5 marks), (b) Name 4 typical microwave transmission lines (2 marks), (c) For TEM mode lines, what is the relationship between electric field, magnetic field and direction propagation? (1 marks) and (d) Sketch electric field lines and magnetic field lines in a coaxial line propagating TEM mode (2 marks).
2. (a) What are the advantages and disadvantages of rectangular waveguide as compared with coaxial line? (4 marks), (b) What are the advantages and disadvantages of circular waveguide as compared with rectangular waveguide? (4 marks) and (c) a_1 and b_1 are incident and scattered wave amplitudes of port 1. a_2 and b_2 are incident and scattered wave amplitudes of port 2. Define S_{21} for two port network (2 marks).

Signals and Systems

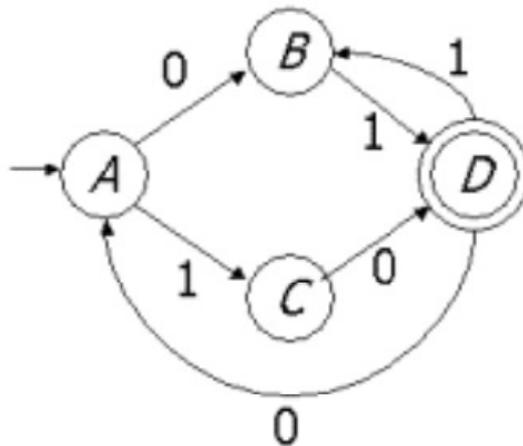
1. Consider the impulse response of FIR filter as $h(n)=\{0.5, 0.5\}$. Whether $h(n)$ is high-pass filter? Clearly justify your answer (10 marks).
2. If continuous-time signal $x(t)$ is known to have bandwidth as 16 kHz and needs to be sampled, then what sampling frequency you will select in order to ensure that exact reconstruction of $x(t)$ is possible? Justify (10 marks)

Digital Signal Processing

1. If $x(n)$ and $h(n)$ are of length 4, respectively, then what will length of their circular convolution (10 marks)?
2. Compare impulse invariant transformation and bilinear transformation for design of IIR digital filter (10 marks).

PhD Entrance Written Test Computer Science

1. Let S and T be strings of lengths m and n respectively. Your task is to determine whether T is a sub-sequence of S , that is, whether all the symbols of T occur in S in the same order as they appear in T , but not necessarily contiguously. For example, the string *grim* is a sub-sequence of the string *algorithm*, whereas the string *gram* is not. Design an $O(m + n)$ time algorithm to solve this problem. Also show that your algorithm runs in $O(m + n)$ time.
2. Consider that you work for a company that manufactures glass jars. Your job is to determine the maximum height from which a jar could be dropped, and still not break. The setup for this experiment is as follows. You have a ladder with n steps, and you want to find the highest step from which you can drop a jar and not have it break. We call this the *highest safe step*. If you have only 1 jar, you could try the following strategy: start by dropping the jar from the first step, then the second step, then the third step and so on until the jar breaks. In this way, you need only one jar, but in the worst case, you may need to drop it n times. Now consider that you are given 2 jars. Design an algorithm to find the *highest safe step*. Your objective is to minimize the number of times you drop jars.
3. Compute the regular expression that defines the same language as the following Deterministic Finite Automaton:



4. Let L be a language over the binary alphabet. The operation $DM(L)$ is defined as follows:
 - Throw away every even-length string from L .
 - For each odd-length string, remove the middle character.

For example, if $L = \{001; 1100; 10101\}$, then $DM(L) = \{01; 1001\}$. That is, the even-length string 1100 is deleted, the middle character of 001 is removed to make 01, and the middle character of 10101 is removed to make 1001.

It turns out that if L is a regular language, then $DM(L)$ may or may not be regular.

For each of the following regular languages L , determine whether or not $DM(L)$ is regular. If the resulting language is regular, then give an equivalent regular expression for it. If the language is not regular, then prove that it is not regular.

- a) L_1 : the language of regular expression $(101)^*$.
- b) L_2 : the language of regular expression 00^*11^*

5. IEEE 802.3 (Ethernet) LAN has length limitations.
 - a. Given other LAN parameters (link capacity, packet size, attenuation rate, propagation speed etc.) derive an expression that must be satisfied for L_{\max} (maximum LAN length)
 - b. Should this limitation also hold on switched Ethernet LAN? Explain.
6. A router is said to be congested if its buffer and packet forwarding resources are inadequate to handle incoming flows for a sustained period. TCP has the mechanism to manage congestion. In this context,
 - a) What is the effect of congestion on the TCP and UDP flows passing through the congested router?
 - b) What is the effect of congestion on the two TCP flows with unequal round trip times passing through the congested router.
7. Suppose a new process in a system arrives at an average of six processes per minute and each such process requires an average of 8 seconds of service time. Estimate the fraction of time the CPU is busy in a system with a single processor.
8. Consider the following relations with underlined primary keys.

Product(P_code, Description, Stocking_date, QtyOnHand, MinQty, Price, Discount, V_code)

Vendor(V_code, Name, Address, Phone)

Here a vendor can supply more than one product but a product is supplied by only one vendor.

Write SQL queries for the following:

- a) List the names of all the vendors who supply more than one product.
 - b) List the details of the products whose prices exceed the average product
9. Given $R(A, B, C, D, E)$ with the set of FDs, $F \{ AB \rightarrow CD, A \rightarrow E, C \rightarrow D \}$. Is the decomposition of R into $R_1(A, B, C)$, $R_2(B, C, D)$ and $R_3(C, D, E)$ lossless? Explain.

10. Assume you have the following jobs to execute with one processor, with the jobs arriving in the order listed here:

i $T(p_i)$

0 80

1 20

2 10

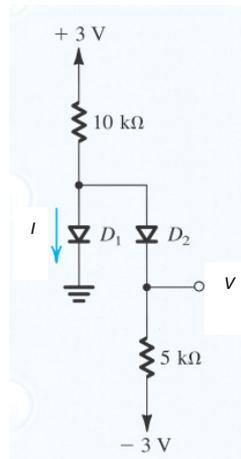
3 20

4 50

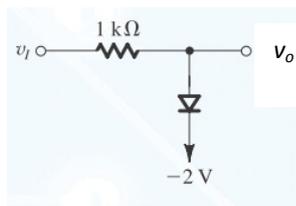
- a) Suppose a system uses Shortest Job Next scheduling. Create a Gantt chart illustrating the execution of these processes?
- b) What is the turnaround time for process p3?
- c) What is the average wait time for the processes?

PhD Entrance Written Test Electronics

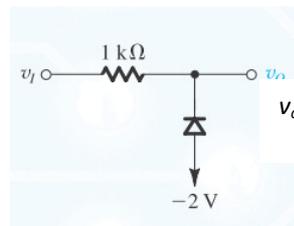
1(a). Assuming that the diodes in the circuits are ideal, find the values of the labeled voltages and currents.



1(b). Sketch the transfer characteristic v_o versus v_i for the limiter circuits shown below. All diodes begin conducting at a forward voltage drop of 0.5 V and have voltage drops of 0.7 V when conducting a current $i_D \geq 1 \text{ mA}$.

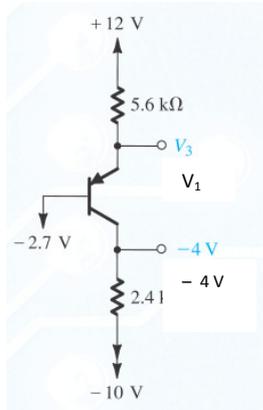


(a)

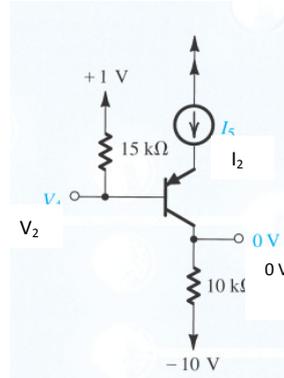


(b)

2. For the circuits shown below, assume that the transistors have very large β . Some measurements have been made on these circuits, with the results indicated in the figure. Find the values of the other labeled voltages and currents.



(a)



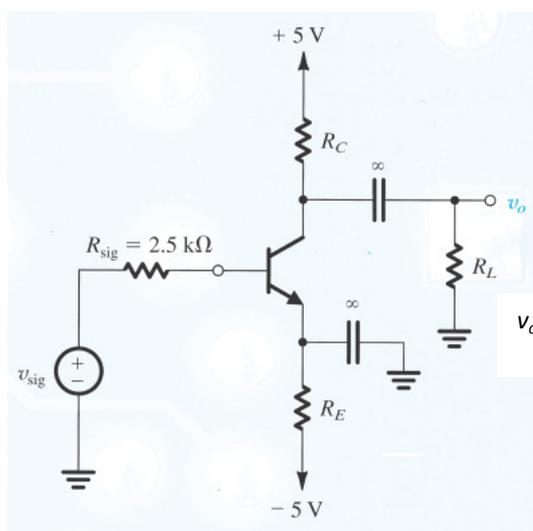
(b)

3. In the circuit shown, v_{sig} is a small sine wave signal with zero average. The transistor β is 100.

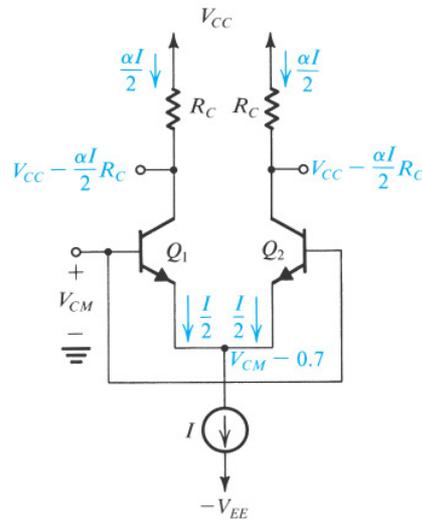
(a) Find the value of R_E to establish a dc emitter current of about 0.5 mA.

(b) Find R_C to establish a dc collector voltage of about +1 V.

(c) For $R_L = 10\text{ k}\Omega$, draw the small-signal equivalent circuit of the amplifier and determine its overall voltage gain.



4. For the differential amplifier of Fig. below let $I = 1 \text{ mA}$, $V_{CC} = 5 \text{ V}$, $v_{CM} = -2 \text{ V}$, $R_C = 3 \text{ k}\Omega$, and $\beta = 100$. Assume that the BJTs have $v_{BE} = 0.7 \text{ V}$ at $i_C = 1 \text{ mA}$. Find the voltage at the emitters and at the outputs.



5. Make circuit of op-amp based non-inverting amplifier circuit, which amplifies a signal $v_{in} = 5 \sin 3t \text{ mV}$ by a factor of 11. Draw the waveforms of input and amplified signals. Mention the values of components used.
6. Explain the working of a successive approximation Analog to Digital Converter (SA-ADC). Make use of timing diagram to show conversion time of a 8 bit SA-ADC.
7. Create a digital timer, which generates an output pulse of one clock period after completion of 5 seconds. Make use of finite state machine controller along with state transition table. Assume that circuit is using clock of 1 Hz. Draw a timing diagram which shows clock, output and states.

PhD Entrance Written Test
English and Philosophical Studies

A. Write a short note on any two of the following (200 words): 10

1. Democracy and Rhetoric
2. *Rasa nishpatti*
3. Modernism
4. Fusion of Horizons
5. Autobiography and the Self
6. Impact of New Media on Literary Studies

B. Write an essay on ONE of the following (1000 words): 25

1. Eminent Marathi writer Bhalchandra Nemade, a Jnanpith awardee, stirred quite a controversy when he questioned the literary merits in the writings of V S Naipaul and Salman Rushdie. He accused them of “pandering to the West”. What are your opinions about the writings of either Rushdie or Naipaul?

OR

In the nineteenth and early twentieth century South Asia the imaginary sense of the public was formed through an understanding of print. The consumption of printed objects made it possible for people to have an awareness that the same objects were being read by a large number of anonymous people. The understanding of a printed object was premised on this awareness, to the point that we now consider it simply axiomatic to consider printing as "publication." In a culture dominated by print, thus understood, whatever one uttered in public was aimed at an audience that was by definition indefinite. Therefore, the consciousness of an invisible, non-individuated audience became an emblem of distinction in print based public culture. How do you assess the cultural transformation engineered by print in South Asian public sphere? And how do you characterize the political character of such a public sphere? Write a short argumentative essay.

2. Examine at least **two** texts that you may have read to discuss how any **one** of the following form has undergone expansion of its generic boundaries, adopted cross-generic properties, or given way to new subgenres in the contemporary period:

a) Novel

b) Drama

c) Poetry

OR

The philosopher Jurgen Habermas claimed that in the wake of aesthetic modernity “the utopian dimension of art, as aesthetic education, disappeared. Aesthetic became a rarefied category that is irreconcilable with the social world. Then the excesses of modernism (avant-garde) in making the antagonism between the aesthetic and the social complete. This is followed by vociferous anti-modernism—to level art and life, fiction and praxis, appearance and reality, the usual symptoms we associate with postmodernism.” Do you agree with this claim? How do you understand the antagonism between the aesthetic and the social in the twentieth century? Write an argumentative answer with appropriate illustrations.

1. Write a critical assessment of the title, theme, diction, imagery and rhyme scheme in the following poem by Adrienne Rich

"Aunt Jennifer's Tigers"

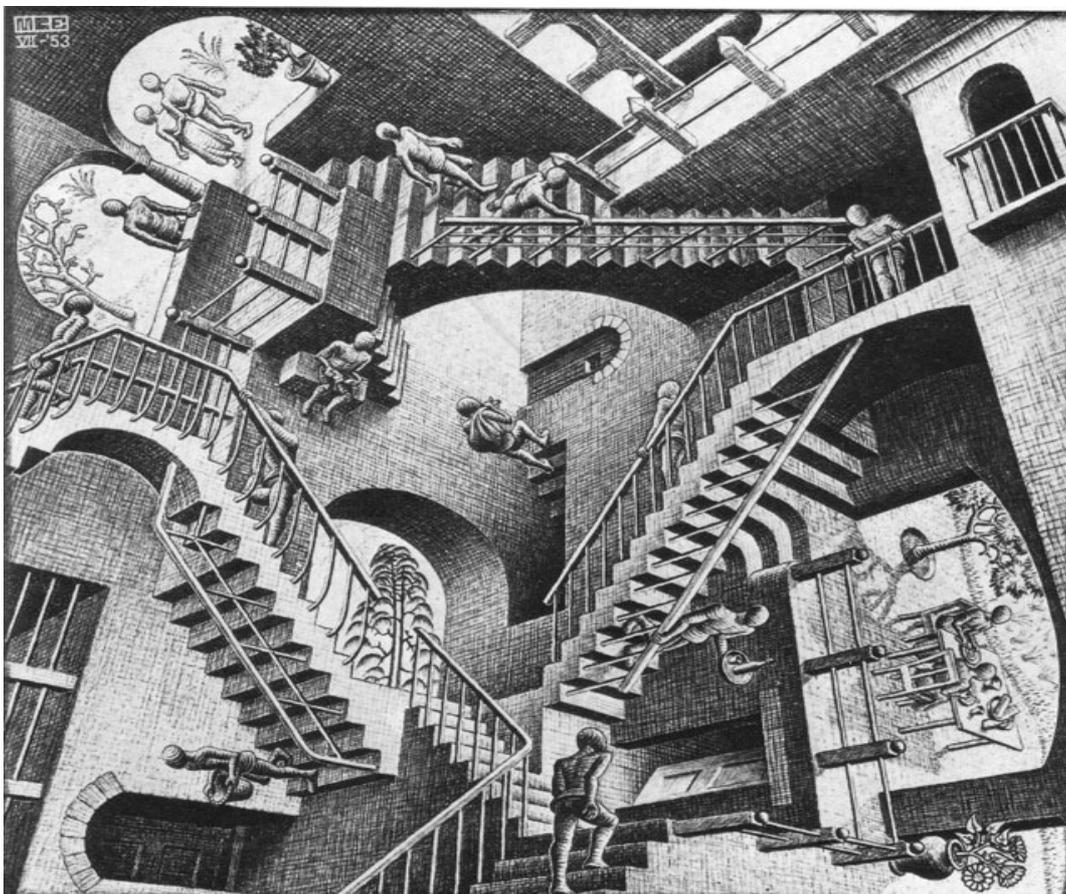
Aunt Jennifer's tigers prance across a screen,
Bright topaz denizens of a world of green.
They do not fear the men beneath the tree;
They pace in sleek chivalric certainty.

Aunt Jennifer's finger fluttering through her wool
Find even the ivory needle hard to pull.
The massive weight of Uncle's wedding band
Sits heavily upon Aunt Jennifer's hand.

When Aunt is dead, her terrified hands will lie
Still ringed with ordeals she was mastered by.
The tigers in the panel that she made
Will go on prancing, proud and unafraid

OR

2. Study the following image etched by M. C. Escher titled "Relativity". Write a coherent descriptive account of the image. What makes the image perplexing and how do you explain its perplexing character?



PhD Entrance Written Test

Economics

1. What should be given more prominence in India: The public sector or private sector? In other words, would you prefer an economy with more state governed enterprises or an economy with predominant presence of private enterprises? Please substantiate your stand with economic arguments. (15 marks)

2. Explain why there is looming agriculture crisis in India at present. (15 marks)

3. Write short notes on the following: (10*2 = 20 marks)
 - a. Indian economy has grown in last 68 years and its structure has also changed. Explain how. Briefly explain whether the change in structure is a normal process or it is worrisome.

 - b. Give the benefits and disadvantages that can be brought in by the foreign direct investment.